Appendix for: "I Alone Can Fix It:" the Strongman Narrative and Leader Support

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A.1 Proof of Proposition 1

Proposition 1. *There exist thresholds $x_0^{SN} \equiv -\gamma$ and $x_1^{SN} \equiv -\gamma(1-\gamma)$, such that there is positive support in equilibrium only if $x \geq x_0^{SN}$. When $x > x_1^{SN}$, in the unique equilibrium, the citizen always supports. When $x \in \left[x_0^{SN}, x_1^{SN}\right]$, there exists an equilibrium where the citizen always supports ($\beta^* = 1$) and one where she supports with interior probability $\beta^* = \beta^I \equiv \frac{x+\gamma(1-\gamma)}{x\gamma}$.*

Using the definition of the net expected utility of supporting, in particular that it is increasing, we have that there is a unique always-support Personal Equilibrium if

$$\text{NE}(\beta = 0, x) > 0 \implies x > -\gamma(1-\gamma) \equiv x_1^{SN}.$$

Moreover, there is a unique never-support equilibrium if

$$\text{NE}(\beta = 1, x) < 0 \implies x < -\gamma \equiv x_0^{SN}.$$

Observe that $x_0^{SN} < x_1^{SN}$.

Finally, there is a unique mixed strategy Personal Equilibrium when

$$\text{NE}(\beta^I, x) = 0 \implies \beta^I = \frac{-x-\gamma(1-\gamma)}{\gamma(-x)}.$$

This is interior if $x > x_0^{SN}$ and $x < x_1^{SN}$.

A.2 Proof of Corollary 1

Corollary 1. *The necessary threshold for support, x_0^{SN}, decreases in γ while the sufficient threshold for support, x_1^{SN}, is minimized at $\gamma = 0.5$.*

Suppose first that $x_0^{SN} = -\gamma$ is the equilibrium threshold. Then:

$$\frac{\partial x_0^{SN}}{\partial \gamma} = -1 < 0.$$

So an increase in γ decreases the threshold x_0^{SN}.

Now suppose that $x_1^{SN} = -\gamma(1-\gamma)$ is the equilibrium threshold. Then:

$$\frac{\partial x_1^{SN}}{\partial \gamma} = 2\gamma - 1$$

So an increase in γ increases the threshold x_1^{SN} if $\gamma > \frac{1}{2}$ and decreases it otherwise.

A.3 Analysis of the *SN* with the General Leader Strength Production Function

Recall from Expression **??** the main text that the general leader strength technology is:

$$\Pr(\theta = 1 \mid a, y) = \alpha_0 + \alpha_a a + \alpha_y y + \alpha_i a y \equiv \alpha(a, y).$$

As indicated, when viewed as a function of a and y, we denote it by α. For the calculations below, an important quantity is the *unconditional* probability of a strong leader, $\Pr(\theta = 1) = \alpha_0 + \alpha_a\beta + \alpha_y\gamma + \alpha_i\gamma\beta$. This is a function of the long-run frequency of support, β.[1]

Beginning with the analysis, the factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_\theta \Pr(\theta \mid a) \Pr(y = 1 \mid \theta).$$

We have:

$$\Pr(\theta = 1 \mid a = 1) = \alpha_0 + \alpha_a + \gamma(\alpha_y + \alpha_i)$$

and $\Pr(\theta = 1 \mid a = 0) = \alpha_0 + \gamma\alpha_y$.

[1] For Case 6. in the proof of Proposition 2, we use the notation $q(\beta) \equiv \alpha_0 + \alpha_a\beta + \alpha_y\gamma + \alpha_i\gamma\beta$ as a shorthand and to emphasize the dependency on β.

Moreover, the probability that the economy is good conditional on a strong leader is:

$$\begin{aligned}\Pr(y=1 \mid \theta=1) &= \frac{\Pr(y=1)\Pr(\theta=1 \mid y=1)}{\Pr(\theta=1)} \\ &= \frac{\gamma\left(\alpha_0+\alpha_y+\beta(\alpha_a+\alpha_i)\right)}{\alpha_0+\alpha_a\beta+\alpha_y\gamma+\alpha_i\beta\gamma}.\end{aligned}$$

Note that if $\alpha_0=\alpha_a=\alpha_y=0$ and $\alpha_i=1$, this simplifies to $\frac{\beta\gamma}{\beta\gamma}=1$ (for $\beta>0$). In general, in order for the expression $\Pr(y=1 \mid \theta=1)$ to be well-defined for all β, it must be the case that $\alpha_0+\alpha_y\gamma>0$, i.e., either α_0 or α_y must be strictly positive.

Similarly, the probability that the economy is good conditional on a weak leader is:

$$\begin{aligned}\Pr(y=1 \mid \theta=0) &= \frac{\Pr(y=1)\Pr(\theta=0 \mid y=1)}{\Pr(\theta=0)} \\ &= \frac{\gamma\left(1-\alpha_0-\alpha_y-\beta(\alpha_a+\alpha_i)\right)}{1-(\alpha_0+\alpha_a\beta+\alpha_y\gamma+\alpha_i\beta\gamma)}.\end{aligned}$$

Note that if $\alpha_0=\alpha_a=\alpha_y=0$ and $\alpha_i=1$, this simplifies to $\frac{\gamma(1-\beta)}{1-\beta\gamma}$.

The expected utility of support is:

$$\Pr(\theta=1 \mid a=1)\Pr(y=1 \mid \theta=1)+\Pr(\theta=0 \mid a=1)\Pr(y=1 \mid \theta=0)+x.$$

The expected utility of not supporting the leader is:

$$\Pr(\theta=1 \mid a=0)\Pr(y=1 \mid \theta=1)+\Pr(\theta=0 \mid a=0)\Pr(y=1 \mid \theta=0).$$

Hence, the citizen chooses $a=1$ if:

$$\underbrace{[\Pr(\theta=1 \mid a=1)-\Pr(\theta=1 \mid a=0)]}_{\text{Effect of Support on Leader Strength}}\underbrace{[\Pr(y=1 \mid \theta=1)-\Pr(y=1 \mid \theta=0)]}_{\text{Effect of Strength on Economy}}+x \geq 0$$

Plugging in, we have that the citizen supports if:

$$\tilde{\text{NE}}(\beta,x) \equiv (\alpha_a+\gamma\alpha_i)\,\frac{\gamma(1-\gamma)(\alpha_y+\beta\alpha_i)}{\Pr(\theta=1)\Pr(\theta=0)}+x \geq 0.$$

If $\alpha_0 = \alpha_a = \alpha_y = 0$ and $\alpha_i = 1$, $\Pr(\theta = 1)\Pr(\theta = 0)$ is equal to $\beta\gamma(1 - \gamma\beta)$, and the net expected utility of support, $\tilde{\text{NE}}$, simplifies to:

$$\frac{\gamma\gamma(1-\gamma)\beta}{\beta\gamma(1-\beta\gamma)} + x = \frac{\gamma(1-\gamma)}{1-\beta\gamma} + x,$$

which is increasing in β (whenever it is well-defined, i.e., $\beta > 0$).

We now show that the function $\tilde{\text{NE}}$ increasing in β if $\alpha_0\alpha_i \geq \alpha_a\alpha_y$, which we assumed. Differentiate the function with respect to β. The derivative is equal to $\frac{(\alpha_a+\alpha_i\gamma)\gamma(1-\gamma)}{\Pr(\theta=1)^2\Pr(\theta=0)^2}$ times:

$$\alpha_i \Pr(\theta = 1)\,(1 - \Pr(\theta = 1)) - (\alpha_y + \alpha_i\beta)\,\frac{\partial \Pr(\theta = 1)}{\partial\beta}(1 - 2\Pr(\theta = 1))$$

Recall that $\frac{\partial \Pr(\theta=1)}{\partial\beta} = \alpha_a + \alpha_i\gamma$. Then, the preceding expression can be re-arranged to obtain:

$$\Pr(\theta = 1)\,[\alpha_i\,(1 - \Pr(\theta = 1)) + 2z] - z,$$

where $z \equiv (\alpha_y + \alpha_i\beta)\,(\alpha_a + \alpha_i\gamma)$. Consider the expression in square parenthesis. We have:

$$\alpha_i\,(1 - \Pr(\theta = 1)) + 2z = \alpha_a\alpha_y + \alpha_i(1 - \alpha_0) + z.$$

Hence, the sign of the derivative is determined by the sign of:

$$\Pr(\theta = 1)\,[\alpha_a\alpha_y + \alpha_i(1 - \alpha_0) + z] - z.$$

This expression is increasing in β and γ (recall that $\Pr(\theta = 1)$ is also an increasing function of β). Hence, it is enough to show that it is positive for $\gamma = \beta = 0$. Plugging in yields:

$$\alpha_0\,[\alpha_a\alpha_y + \alpha_i(1 - \alpha_0) + \alpha_a\alpha_y] - \alpha_a\alpha_y,$$

or

$$\alpha_0\alpha_a\alpha_y + (1 - \alpha_0)\,[\alpha_0\alpha_i - \alpha_a\alpha_y]\,.$$

This is positive by the assumption that $\alpha_0\alpha_i \geq \alpha_a\alpha_y$.

Thus, similar to the baseline case, there are two thresholds $\tilde{x}_0^{SN}$ and $\tilde{x}_1^{SN}$ such that there is a unique always support equilibrium if $x > \tilde{x}_1^{SN}$ and a unique never support equilibrium if $x < \tilde{x}_0^{SN}$. Importantly, since $\tilde{\text{NE}}$ evaluated at $x = 0$ is positive, both of these thresholds are (weakly) negative.

In particular, the necessity $\tilde{x}_0^{SN}$ and sufficiency $\tilde{x}_1^{SN}$ thresholds are defined when β is equal to 1 and 0, respectively, i.e.:

$$\tilde{\text{NE}}\left(1, \tilde{x}_0^{SN}\right) = 0 \quad \text{and} \quad \tilde{\text{NE}}\left(0, \tilde{x}_1^{SN}\right) = 0. \tag{1}$$

In explicit terms, the critical values in Expression 1 are as follows:

$$\begin{aligned}
\tilde{x}_0^{SN} &= -\left(\alpha_a + \alpha_i\gamma\right)\frac{\gamma(1-\gamma)\left(\alpha_y + \alpha_i\right)}{\left(\alpha_0 + \alpha_a + \gamma(\alpha_y + \alpha_i)\right)\left(1 - \alpha_0 - \alpha_a - \gamma(\alpha_y + \alpha_i)\right)} \\
\tilde{x}_1^{SN} &= -\left(\alpha_a + \alpha_i\gamma\right)\frac{\gamma(1-\gamma)\alpha_y}{\left(\alpha_0 + \gamma\alpha_y\right)\left(1 - \alpha_0 - \gamma\alpha_y\right)}
\end{aligned}$$

Here, we already plugged in $\Pr(\theta = 1)|_{\beta=1} = \alpha_0 + \alpha_a + \gamma(\alpha_y + \alpha_i)$ and $\Pr(\theta = 1)|_{\beta=0} = \alpha_0 + \gamma\alpha_y$. Note that because the function $\tilde{\text{NE}}$ is increasing in β, $\tilde{x}_0^{SN} < \tilde{x}_1^{SN}$.

Comparison to Benchmark Case Observe that the benchmark case is qualitatively different because $\Pr(y = 1 \mid \theta = 1) = 1$ (for $\beta > 0$) is not a function of β. In general, $\Pr(y = 1 \mid \theta = 1) = \frac{\gamma(\alpha_0+\alpha_y+\beta(\alpha_a+\alpha_i))}{\alpha_0+\alpha_y\gamma+\alpha_a\beta+\alpha_i\beta\gamma}$ is an increasing function of β, as shown in Figure 1. The left panel shows the case in which $\alpha_y = 0$, so the difference in posterior beliefs is also 0 when $\beta = 0$. As a result, the sufficiency threshold $\tilde{x}_1^{SN}$ is then 0.

A.4 Proof of Proposition 2

Proposition 2. *Suppose leader strength is produced according to Equation* (**??**). *Then the SN is the most desirable narrative for the leader among all narratives with three nodes a, θ, and y.*

Recall that $\alpha(a, y) = \alpha_0 + \alpha_a a + \alpha_y y + \alpha_i ay$ is the probability of a strong leader

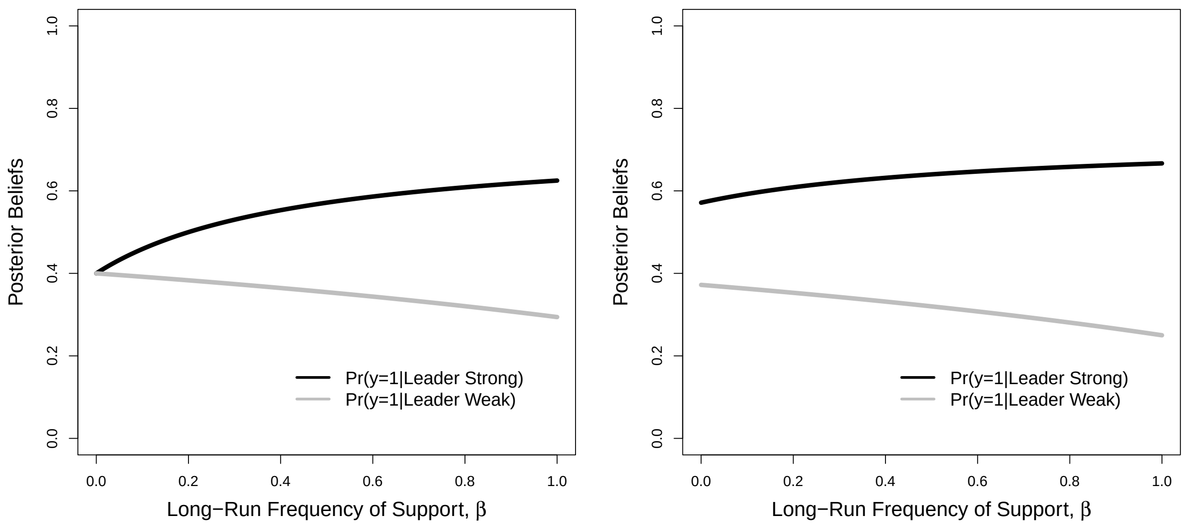

Figure 1: Posterior Beliefs $\Pr(y = 1 \mid \theta = 1)$ and $\Pr(y = 1 \mid \theta = 0)$. Parameter values: $\alpha_0 = 0.1$, $\alpha_a = 0.1$, $\alpha_i = 0.3$, $\gamma = 0.4$. Left panel: $\alpha_y = 0$, right panel: $\alpha_y = 0.1$

conditional on support a and economic performance y. In principle, there are 10 possible narratives to check; however, only six of them are plausible candidates for an optimal narrative. We can rule out the narratives $a \rightarrow \theta \quad y$, $a \quad \theta \leftarrow y$, $a \quad \theta \rightarrow y$, $a \rightarrow \theta \leftarrow y$ because none has a path—direct or indirect—that goes from a to y. This means that the citizen believes her action cannot have an effect on the economy and thus will only support if x is positive.

We first solve for the cases where a is the only cause of y. Notice that these narratives also cannot be optimal, because the conditional probability $\Pr(y \mid a)$ reveals that a and y are independent.

Case 1. Suppose that the narrative is $a \rightarrow \theta \leftarrow y$ and $a \rightarrow y$ (i.e., the narrative corresponding to the actual data generating process plus an arrow from a to y). The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta} \Pr(\theta \mid a, y = 1) \Pr(y = 1 \mid a).$$

Plugging in terms, the expected utility of $a = 1$ is $\alpha(1,1)\gamma + (1 - \alpha(1,1))\gamma + x = \gamma + x$ whereas the expected utility of $a = 0$ is $\alpha(0,1)\gamma + (1 - \alpha(0,1))\gamma = \gamma$. Hence, the citizen supports if $x \geq 0$.

Case 2. Suppose that the narrative is $y \leftarrow a \rightarrow \theta$. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta} \Pr(\theta \mid a) \Pr(y = 1 \mid a).$$

This is just $\Pr(y = 1 \mid a) = \gamma$, independent of a. Hence, the citizen supports if $x \geq 0$.

Case 3. Suppose that the narrative is $a \rightarrow y \rightarrow \theta$. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta} \Pr(y = 1 \mid a) \Pr(\theta \mid y)$$

This can be re-arranged to $\Pr(y = 1 \mid a) \sum_{\theta} \Pr(\theta \mid y)$ which is just $\Pr(y = 1 \mid a) = \gamma$ for any a. Hence, the citizen supports if $x \geq 0$. .

This leaves us with three narratives: the *SN*, and two similar narratives that are its modifications. We start with the *SN*.

Case 4. Suppose that the narrative is $a \rightarrow \theta \rightarrow y$ (i.e., the *SN*). This is analyzed in section A.3 above. We show that similar to the baseline case, there are two thresholds $\tilde{x}_0^{SN} < 0$ and $\tilde{x}_1^{SN} \leq 0$ (with strict inequality if $\alpha_y > 0$) such that there is a unique always support equilibrium if $x > \tilde{x}_1^{SN}$ and a unique never support equilibrium if $x < \tilde{x}_0^{SN}$. Since $\tilde{\text{NE}}$ evaluated at $x = 0$ is positive, both of these thresholds are (weakly) negative. Therefore, the *SN* is more desirable for the leader according to Definition **??**.

Case 5. Suppose that the narrative is $a \rightarrow \theta \rightarrow y$ and $a \rightarrow y$ (i.e., the *SN* with an additional link from a to y). The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta} \Pr(\theta \mid a) \Pr(y = 1 \mid \theta, a).$$

Plugging in terms, the expected utility of $a = 1$ is

$$\Pr(\theta = 1 \mid a = 1) \Pr(y = 1 \mid \theta = 1, a = 1) + \Pr(\theta = 0 \mid a = 1) \Pr(y = 1 \mid \theta = 0, a = 1) + x.$$

Using the definition of conditional probability, this is equal to:

$$\Pr(\theta = 1 \mid a = 1) \frac{\Pr(y = 1 \mid a = 1) \Pr(\theta = 1 \mid y = 1, a = 1)}{\Pr(\theta = 1 \mid a = 1)} + \\ \Pr(\theta = 0 \mid a = 1) \frac{\Pr(y = 1 \mid a = 1) \Pr(\theta = 0 \mid y = 1, a = 1)}{\Pr(\theta = 0 \mid a = 1)} + x.$$

Canceling yields:

$$\begin{aligned} &\Pr(y = 1 \mid a = 1) [\Pr(\theta = 1 \mid y = 1, a = 1) + \Pr(\theta = 0 \mid y = 1, a = 1)] + x \\ =& \Pr(y = 1 \mid a = 1) + x \\ =& \gamma + x. \end{aligned}$$

Using identical steps but conditioning on $a = 0$ shows that the expected utility of $a = 0$ is $\Pr(y = 1 \mid a = 1) = \gamma$. Hence, the citizen supports if $x \geq 0$.

Case 6. Suppose that the narrative is $a \to y \leftarrow \theta$. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_\theta \Pr(\theta) \Pr(y = 1 \mid \theta, a).$$

Let $q_{a\theta}$ be the posterior probability that $y = 1$ conditional on a and θ. Then, the citizen supports if:

$$\Pr(\theta = 1) q_{11} + (1 - \Pr(\theta = 1)) q_{10} + x \geq \Pr(\theta = 1) q_{01} + (1 - \Pr(\theta = 1)) q_{00}.$$

Re-arranging:

$$\Pr(\theta = 1) [q_{11} - q_{01} - (q_{10} - q_{00})] + q_{10} - q_{00} + x \geq 0.$$

The expression in square brackets is a difference-in-differences estimator. To simplify notation even more, define $e_\theta \equiv q_{1\theta} - q_{0\theta}$ as the effect of a when regime strength is θ. The citizen supports if:

$$\Pr(\theta = 1)(e_1 - e_0) + e_0 + x \geq 0$$

Here, we first prove the following claim:

Result 1. $e_1 \geq 0$ *and* $e_0 < 0$.

Proof. Let $e_1 \equiv \Pr(y = 1 \mid a = 1, \theta = 1) - \Pr(y = 1 \mid a = 0, \theta = 1)$. We have:

$$e_1 = \frac{\gamma\alpha(1,1)}{\gamma\alpha(1,1) + (1-\gamma)\alpha(1,0)} - \frac{\gamma\alpha(0,1)}{\gamma\alpha(0,1) + (1-\gamma)\alpha(0,0)}.$$

We wish to show that $e_1 \geq 0$. To prove this, suppose not, i.e., $e_1 < 0$. This is the same as:

$$\frac{\gamma\alpha(1,1)\left[\gamma\alpha(0,1) + (1-\gamma)\alpha(0,0)\right] - \gamma\alpha(0,1)\left[\gamma\alpha(1,1) + (1-\gamma)\alpha(1,0)\right]}{\Pr(\theta = 1 \mid a = 1)\Pr(\theta = 1 \mid a = 0)} < 0.$$

Simplifying yields:

$$\gamma\alpha(1,1)\alpha(0,1) + (1-\gamma)\alpha(1,1)\alpha(0,0) - \gamma\alpha(1,1)\alpha(0,1) - (1-\gamma)\alpha(1,0)\alpha(0,1) < 0,$$

or:

$$(1-\gamma)\left[\alpha(1,1)\alpha(0,0) - \alpha(1,0)\alpha(0,1)\right] < 0.$$

And finally:

$$\alpha(1,1)\alpha(0,0) < \alpha(1,0)\alpha(0,1).$$

Plugging in terms:

$$\alpha_0\alpha_i < \alpha_a\alpha_y.$$

But this contradicts the assumption that $\alpha_0\alpha_i \geq \alpha_a\alpha_y$. Hence, $e_1 \geq 0$.

Next, we have $e_0 \equiv \Pr(y=1 \mid a=1, \theta=0) - \Pr(y=1 \mid a=0, \theta=0)$, or

$$e_0 = \frac{\gamma(1-\alpha(1,1))}{\gamma(1-\alpha(1,1)) + (1-\gamma)(1-\alpha(1,0))} - \frac{\gamma(1-\alpha(0,1))}{\gamma(1-\alpha(0,1)) + (1-\gamma)(1-\alpha(0,0))}.$$

We wish to show that $e_0 < 0$. Suppose to the contrary: $e_0 \geq 0$. It follows that:

$$\frac{\gamma(1-\alpha(1,1))\left[\gamma(1-\alpha(0,1)) + (1-\gamma)(1-\alpha(0,0))\right]}{\Pr(\theta=0 \mid a=1)\Pr(\theta=0 \mid a=0)} - \frac{\gamma(1-\alpha(0,1))\left[\gamma(1-\alpha(1,1)) + (1-\gamma)(1-\alpha(1,0))\right]}{\Pr(\theta=0 \mid a=1)\Pr(\theta=0 \mid a=0)} \geq 0.$$

Simplifying yields:

$$(1-\gamma)\left[(1-\alpha(1,1))(1-\alpha(0,0)) - (1-\alpha(1,0))(1-\alpha(0,1))\right] \geq 0.$$

And finally:

$$\alpha(0,1) + \alpha(1,0) - \alpha(1,1) - \alpha(0,0) + \alpha(1,1)\alpha(0,0) \geq \alpha(1,0)\alpha(0,1).$$

Plugging in, we have that $\alpha(0,1) + \alpha(1,0) - \alpha(1,1) - \alpha(0,0) = -\alpha_i$. Therefore:

$$\alpha_i(\alpha_0 - 1) \geq \alpha_a \alpha_y.$$

The left-hand side is always strictly negative while the right-hand side is weakly positive—a contradiction. Hence, $e_0 < 0$. □

As a result, historical complementarity holds because $\Pr(\theta=1)$ is increasing in β (given $\alpha_i > 0$)) and $e_1 - e_0 > 0$. Thus, similar to the case in which the citizen believes in the *SN*, there are thresholds x_0^{WT} and x_1^{WT} such that the citizen always supports if $x > x_1^{WT}$ and never supports if $x < x_0^{WT}$. When $x \in \left[x_0^{WT}, x_1^{WT}\right]$, multiple equilibria exist. However, we show next that the WT narrative eats into the support base of the leader, i.e., $x_0^{WT} > 0$. To see this, utilize the notation in which $q(\beta)$ is the (unconditional) probability of a strong leader as a function of β. Then, we can derive x_0^{WT} as follows:

$$x_0^{WT} = -q(1)(e_1 - e_0) - e_0 = q(1)(-e_1) + (1 - q(1))(-e_0).$$

Plugging in the terms e_1 and e_0, we have:

$$-q(1)\left(q_{11} - q_{01}\right) - (1 - q(1))\left(q_{10} - q_{00}\right).$$

and further plugging in:

$$-q(1)\left(\frac{\gamma\alpha(1,1)}{q(1)} - \frac{\gamma\alpha(0,1)}{q(0)}\right) - (1 - q(1))\left(\frac{\gamma(1 - \alpha(1,1))}{1 - q(1)} - \frac{\gamma(1 - \alpha(0,1))}{1 - q(0)}\right).$$

This can be simplified to obtain:

$$-\gamma\alpha(1,1) + \frac{\gamma\alpha(0,1)q(1)}{q(0)} - \gamma(1 - \alpha(1,1)) - \frac{\gamma(1 - \alpha(0,1))(1 - q(1))}{1 - q(0)}.$$

Rewrite this as follows:

$$-\gamma + \alpha(0,1)\left[\frac{\gamma q(1)}{q(0)} - \frac{\gamma(1 - q(1))}{1 - q(0)}\right] + \frac{\gamma(1 - q(1))}{1 - q(0)}.$$

Finally, we have:

$$\frac{\gamma q(0)\left[q(0) - q(1)\right] + \gamma\alpha(0,1)\left[q(1) - q(0)\right]}{q(0)(1 - q(0))},$$

or

$$\frac{\gamma\left[q(1) - q(0)\right]\left[\alpha(0,1) - q(0)\right]}{q(0)(1 - q(0))}.$$

Plugging in $\alpha(0,1) = \alpha_0 + \alpha_y$ and $q(0) = \alpha_0 + \alpha_y\gamma$, we have:

$$x_0^{WT} = \frac{\gamma(1 - \gamma)\alpha_y\left[q(1) - q(0)\right]}{q(0)(1 - q(0))}.$$

For the expression to be well defined, it needs to be the case that $q(0) \in (0,1)$ (otherwise, as noted in the main text, the probabilities q_{01} and q_{00} are undefined, conditioning on probability 0 events). This requires that $\alpha_0 + \alpha_y \gamma \in (0, 1)$, necessitating either α_0 or α_y to be strictly positive.

Further, observe that $q(1) - q(0) = \alpha_a + \gamma \alpha_i$ is strictly positive because $\alpha_i > 0$ and $\gamma \in (0, 1)$. Hence, provided that x_0^{WT} is well defined, it is weakly positive, and strictly positive if $\alpha_y > 0$.

As a result of this analysis, the WT narrative is (weakly) less desirable than the BO narrative according to Definition **??** (strictly so if $\alpha_y > 0$). Therefore, the WT narrative cannot be the most desirable for the leader.

A.5 Proof of Proposition 3

Proposition 3. *There exist thresholds $\tilde{x}^{SN}_{\lambda,0}$ and $\tilde{x}^{SN}_{\lambda,1}$, such that there is positive support in equilibrium only if $x \geq \tilde{x}^{SN}_{\lambda,0}$. When $x > \tilde{x}^{SN}_{\lambda,1}$, in the unique equilibrium, the citizen always supports. When $x \in [\tilde{x}^{SN}_{\lambda,0}, \tilde{x}^{SN}_{\lambda,1}]$, there exists an equilibrium where the citizen always supports ($\beta^* = 1$) and one where she supports with some interior probability.*

The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta,c} \Pr(\theta \mid a) \Pr(c \mid \theta) \Pr(y = 1 \mid \theta).$$

Moreover, the marginal probability of $c = 1$, conditional on a, is:

$$\sum_{\theta,y} \Pr(\theta \mid a) \Pr(c = 1 \mid \theta) \Pr(y \mid \theta).$$

Given our assumption on the probability of the consequence c, the probability of high economic performance when the citizen believes in the SN simplifies to

$$\Pr(\theta = 1 \mid a) \Pr(y = 1 \mid \theta = 1) + \Pr(\theta = 0 \mid a) \Pr(y = 1 \mid \theta = 0).$$

This is the same expression as in the case in which the variable c is omitted. Hence, following previous calculations, the difference in the probability of obtaining high economic performance when the citizen supports and does not support is equal to:

$$[\Pr(\theta = 1 \mid a = 1) - \Pr(\theta = 1 \mid a = 0)] \, [\Pr(y = 1 \mid \theta = 1) - \Pr(y = 1 \mid \theta = 0)] \, .$$

Plugging in, this expression is equal to:

$$(\alpha_a + \alpha_i \gamma) \frac{\gamma(1-\gamma)\,(\alpha_y + \alpha_i \beta)}{\Pr(\theta = 1)\,(1 - \Pr(\theta = 1))}.$$

Now consider how the probability of the consequence c changes with a. If $a = 1$, then the citizen expects $c = 1$ with probability $(\alpha_0 + \alpha_a + \alpha_y \gamma + \alpha_i \gamma)\, q$. If $a = 0$, then the citizen expects the consequence with probability $(\alpha_0 + \alpha_y \gamma)\, q$.

Putting everything together, the net expected utility of supporting the leader is:

$$\tilde{\text{NE}}_\lambda(\beta, x) = (\alpha_a + \alpha_i \gamma) \frac{\gamma(1-\gamma)\,(\alpha_y + \alpha_i \beta)}{\Pr(\theta = 1)\,(1 - \Pr(\theta = 1))} + (\alpha_a + \alpha_i \gamma) q \lambda + x.$$

Observe that the term $(\alpha_a + \alpha_i \gamma) \frac{\gamma(1-\gamma)(\alpha_y + \alpha_i \beta)}{\Pr(\theta=1)(1-\Pr(\theta=1))}$ is positive. Moreover, the term $(\alpha_a + \alpha_i \gamma)\lambda$ is positive if $\lambda > 0$ and negative otherwise.

Hence, there are thresholds $\tilde{x}^{SN}_{\lambda,0}$ and $\tilde{x}^{SN}_{\lambda,1}$ such that there is support in equilibrium only if $x \geq \tilde{x}^{SN}_{\lambda,0}$. In fact, the citizen always supports if $x > \tilde{x}^{SN}_{\lambda,1}$. When $x \in \left[\tilde{x}^{SN}_{\lambda,0}, \tilde{x}^{SN}_{\lambda,1}\right]$, there is a Personal Equilibrium in which the citizen always supports and one in which the citizen support with strictly interior probability. When $x \leq \tilde{x}^{SN}_{\lambda,1}$, there is an equilibrium in which the citizen never supports, and this is unique if $x < \tilde{x}^{SN}_{\lambda,0}$.

A.6 Proof of Proposition 4

Proposition 4. *Consider the class of narratives that are at most one link away from the true data generating process. There are critical values $\overline{\lambda}_0$ and $\overline{\lambda}_1$, with $\overline{\lambda}_0 < \overline{\lambda}_1 < 0$, such that:*

- *when $\lambda > \overline{\lambda}_1$, the SN is the most desirable narrative for the leader.*
- *when $\lambda \in \left(\overline{\lambda}_0, \overline{\lambda}_1\right)$ and $\tilde{x}^{SN}_{0,\lambda}$ is the selected equilibrium threshold, the SN is the most desirable for the leader.*

- *when* $\lambda \in \left(\overline{\lambda}_0, \overline{\lambda}_1\right)$ *and* $\tilde{x}^{SN}_{1,\lambda}$ *is the selected equilibrium threshold, a narrative that portrays c as exogenous is most desirable for the leader.*
- *when* $\lambda < \overline{\lambda}_0$*, a narrative that portrays c as exogenous is most desirable for the leader.*

As before, denote by $\Pr(\theta = 1 \mid a, y) \equiv \alpha(a, y) = \alpha_0 + \alpha_a a + \alpha_y y + \alpha_i a y$ the probability of a strong leader conditional on support a and economic performance y.

Rational Expectations Consider first the case when the citizen has rational expectations, i.e., believes in the *BO* narrative. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta,c} \Pr(\theta \mid a, y = 1) \Pr(c \mid \theta) \Pr(y = 1).$$

This is equal to γ for any a.

Moreover, the marginal probability of $c = 1$, conditional on a, is:

$$\sum_{\theta,y} \Pr(\theta \mid a, y) \Pr(c = 1 \mid \theta) \Pr(y),$$

This is equal to $(\alpha_0 + \alpha_a + \alpha_y\gamma + \alpha_i\gamma)\, q$ if $a = 1$ and equal to $(\alpha_0 + \alpha_y\gamma)\, q$ if $a = 0$.

Hence, the expected utility of $a = 1$ is $\gamma + x + (\alpha_0 + \alpha_a + \alpha_y\gamma + \alpha_i\gamma)\, q\lambda$. The expected utility of $a = 0$ is $\gamma + (\alpha_0 + \alpha_y\gamma)\, q\lambda$. The citizen supports if:

$$x \geq -\left(\alpha_a + \alpha_i\gamma\right) q\lambda.$$

The right-hand side denotes the expected costs of not matching the support action to the compliance of the non-strategic citizens or benefits of avoiding backsliding. The steady-state probability of support is $\beta^* = \mathbb{1}\left(x \geq -\left(\alpha_a + \alpha_i\gamma\right) q\lambda\right)$.

There are eight additional potential narratives to check, four resulting from adding a link, two from removing a link, and two from inverting a link:

Case 1., Addition 1. Suppose that link $a \to y$ is added. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta,c} \Pr(\theta \mid a, y = 1) \Pr(c \mid \theta) \Pr(y = 1 \mid a).$$

Observe that $\Pr(y = 1 \mid a)$ does not depend on θ or c, so this is just $\Pr(y = 1 \mid a) = \gamma$. Moreover, the marginal probability of $c = 1$, conditional on a, is:

$$\sum_{\theta,y} \Pr(\theta \mid a, y) \Pr(c = 1 \mid \theta) \Pr(y \mid a).$$

Since $\Pr(y \mid a) = \Pr(y)$, this is the same as in the benchmark case with rational expectations. The citizen supports if $x \geq -(\alpha_a + \alpha_i \gamma) q\lambda$.

Case 2., Addition 2. Suppose that link $a \to c$ is added. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta,c} \Pr(\theta \mid a, y = 1) \Pr(c \mid \theta, a) \Pr(y = 1).$$

Similar to Case 1., this is just $\Pr(y = 1) = \gamma$. Moreover, the marginal probability of $c = 1$, conditional on a, is:

$$\sum_{\theta,y} \Pr(\theta \mid a, y) \Pr(c = 1 \mid \theta, a) \Pr(y).$$

When $a = 1$, this is equal to $(\alpha_0 + \alpha_a + \alpha_y \gamma + \alpha_i \gamma) q$. When $a = 0$, this is equal to $(\alpha_0 + \alpha_y \gamma) q$. Hence, the optimal decision rule is the same as in the case of rational expectations.

Case 3., Addition 3. Suppose that link $y \to c$ is added. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta,c} \Pr(\theta \mid a, y = 1) \Pr(c \mid \theta, y = 1) \Pr(y = 1).$$

This is just $\Pr(y = 1) = \gamma$. Moreover, the marginal probability of $c = 1$, conditional on a, is:

$$\sum_{\theta,y} \Pr(\theta \mid a, y) \Pr(c = 1 \mid \theta, y) \Pr(y).$$

Plugging in terms, when $a = 1$, this is equal to $(\alpha_0 + \alpha_a + \alpha_y\gamma + \alpha_i\gamma)\, q$. When $a = 0$, this is equal to $(\alpha_0 + \alpha_y\gamma)\, q$. Hence, the optimal decision rule is the same as in the benchmark case of rational expectations.

Case 4., Addition 4. Suppose that link $c \to y$ is added. This results in a cycle $\theta \to c \to y \to \theta$, rendering this case inadmissible.

Case 5., Removal 1. Suppose that link $y \to \theta$ is removed. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta,c} \Pr(\theta \mid a) \Pr(c \mid \theta) \Pr(y = 1).$$

This is just $\Pr(y = 1) = \gamma$. Moreover, the marginal probability of $c = 1$, conditional on a, is:

$$\sum_{\theta,y} \Pr(\theta \mid a) \Pr(c = 1 \mid \theta) \Pr(y).$$

Plugging in terms, this is equal to $(\alpha_0 + \alpha_a + \alpha_y\gamma + \alpha_i\gamma)\, q$ if $a = 1$ and $(\alpha_0 + \alpha_y\gamma)\, q$ if $a = 0$. Hence, the citizen's decision rule is as in Case 1.

Case 6., Removal 2. Suppose that link $\theta \to c$ is removed. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta,c} \Pr(\theta \mid a) \Pr(c) \Pr(y = 1).$$

This is just $\Pr(y = 1) = \gamma$. Moreover, the marginal probability of $c = 1$, conditional on a, is:

$$\sum_{\theta,y} \Pr(\theta \mid a) \Pr(c = 1) \Pr(y).$$

This is equal to $\Pr(c = 1) = \Pr(\theta = 1) = (\alpha_0 + \alpha_a\beta + \alpha_y\gamma + \alpha_i\beta\gamma)\, q$, independent of a. Hence, the citizen supports if $x \geq 0$.

Case 7., Inversion 1. Suppose that link $y \to \theta$ is inverted. This is the *SN*. As shown in the proof of Proposition **??**, there are thresholds $\tilde{x}^{SN}_{\lambda,0}$ and $\tilde{x}^{SN}_{\lambda,1}$ such that the citizen always supports if $x > \tilde{x}^{SN}_{\lambda,1}$ and never supports if $x < \tilde{x}^{SN}_{\lambda,0}$. When $x \in \left[\tilde{x}^{SN}_{\lambda,0}, \tilde{x}^{SN}_{\lambda,1}\right]$, there are multiple steady state probabilities of support, including $\beta^* = 0$ and $\beta^* = 1$.

In particular, the necessity $\tilde{x}^{SN}_{\lambda,0}$ and sufficiency $\tilde{x}^{SN}_{\lambda,1}$ thresholds are defined when β is equal to 1 and 0, respectively, i.e.:

$$\tilde{\text{NE}}_\lambda \left(1, \tilde{x}^{SN}_{\lambda,0}\right) = 0 \quad \text{and} \quad \tilde{\text{NE}}_\lambda \left(0, \tilde{x}^{SN}_{\lambda,1}\right) = 0, \tag{2}$$

where recall that the net expected utility of supporting under the *SN* is equal to:

$$\tilde{\text{NE}}_\lambda(\beta, x) = (\alpha_a + \alpha_i\gamma) \frac{\gamma(1-\gamma)\,(\alpha_y + \alpha_i\beta)}{\Pr(\theta = 1)\Pr(\theta = 0)} + (\alpha_a + \alpha_i\gamma)\, q\lambda + x.$$

In explicit terms, recalling that $\Pr(\theta = 1)$ is a function of β, the critical values in Expression 2 are as follows:

$$\tilde{x}^{SN}_{\lambda,0} = -\,(\alpha_a + \alpha_i\gamma) \frac{\gamma(1-\gamma)\,(\alpha_y + \alpha_i)}{(\alpha_0 + \alpha_a + \gamma(\alpha_y + \alpha_i))\,(1 - \alpha_0 - \alpha_a - \gamma(\alpha_y + \alpha_i))} - (\alpha_a + \alpha_i\gamma)\, q\lambda,$$
$$\tilde{x}^{SN}_{\lambda,1} = -\,(\alpha_a + \alpha_i\gamma) \frac{\gamma(1-\gamma)\alpha_y}{(\alpha_0 + \gamma\alpha_y)\,(1 - \alpha_0 - \gamma\alpha_y)} - (\alpha_a + \alpha_i\gamma)\, q\lambda.$$

Because the function $\tilde{\text{NE}}_\lambda$ is increasing in β, $\tilde{x}^{SN}_{\lambda,0} < \tilde{x}^{SN}_{\lambda,1}$.

Case 8., Inversion 2. Suppose that link $\theta \to c$ is inverted. This is the DAG in the left panel of Figure **??**. The factorization formula implies that the marginal probability of $y = 1$, conditional on a, is:

$$\sum_{\theta,c} \Pr(\theta \mid a, c, y = 1) \Pr(c) \Pr(y = 1).$$

This is equal to $\Pr(y = 1) = \gamma$. Moreover, the marginal probability of $c = 1$, conditional on a, is:

$$\sum_{\theta,y} \Pr(\theta \mid a, y, c = 1) \Pr(c = 1) \Pr(y).$$

This is just $\Pr(c = 1) = q \Pr(\theta = 1) = q (\alpha_0 + \alpha_a\beta + \alpha_y\gamma + \alpha_i\gamma\beta)$, independent of a. Hence, the citizen supports if:

$$\gamma + x + q \Pr(\theta = 1)\lambda \geq \gamma + q \Pr(\theta = 1)\lambda.$$

Thus, the optimal decision rule is to support whenever $x \geq 0$ (the same as in Case 6.).

Summarizing, we have shown that:

1. Narratives in which c is exogenous (cases 6. and 8.) induce the citizen to support if $x \geq 0$.
2. The *SN* (case 7.) induces the citizen to support if $x \geq \tilde{x}^{SN}_\lambda$, where $\tilde{x}^{SN}_\lambda \in \{\tilde{x}^{SN}_{\lambda,0}, \tilde{x}^{SN}_{\lambda,1}\}$, and both the necessity threshold $\tilde{x}^{SN}_{\lambda,0}$ and sufficiency threshold $\tilde{x}^{SN}_{\lambda,1}$ for the *SN* are both linearly decreasing in λ.
3. All other feasible narratives (rational expectations, cases 1.-3., 5.) induce the citizen to support if $x \geq -(\alpha_a + \alpha_i\gamma) q\lambda$.

First, since $(\alpha_a + \alpha_i\gamma) \frac{\gamma(1-\gamma)(\alpha_y+\alpha_i\beta)}{\Pr(\theta=1)(1-\Pr(\theta=1))} > 0$ for any β, the *SN* is more desirable for the leader than any narrative covered under item 3. above.

Second, define $\overline{\lambda}_0$ ($\overline{\lambda}_1$) as the value of λ at which $\tilde{x}^{SN}_{\lambda,0}$ ($\tilde{x}^{SN}_{\lambda,1}$) is equal to 0. In explicit terms:

$$\overline{\lambda}_0 = -\frac{\gamma(1-\gamma)\left(\alpha_y+\alpha_i\right)}{q\left(\alpha_0+\alpha_a+\gamma(\alpha_y+\alpha_i)\right)\left(1-\alpha_0-\alpha_a-\gamma(\alpha_y+\alpha_i)\right)},$$
$$\overline{\lambda}_1 = -\frac{\gamma(1-\gamma)\alpha_y}{q\left(\alpha_0+\gamma\alpha_y\right)\left(1-\alpha_0-\gamma\alpha_y\right)}.$$

And by inspection, we have that $\overline{\lambda}_0 < \overline{\lambda}_1 < 0$.

By construction:

When $\lambda > \overline{\lambda}_1$, then both $\tilde{x}^{SN}_{\lambda,0}$ and $\tilde{x}^{SN}_{\lambda,1}$ are negative. As a result, the *SN* is the most desirable narrative for the leader.

When $\lambda \in \left(\overline{\lambda}_0, \overline{\lambda}_1\right)$ and $\tilde{x}^{SN}_{\lambda,0}$ is selected, then the *SN* is the most desirable narrative for the leader.

When $\lambda \in \left(\overline{\lambda}_0, \overline{\lambda}_1\right)$ and $\tilde{x}^{SN}_{\lambda,1}$ is selected, then a narrative in which c is exogenous is the most desirable for the leader.

When $\lambda < \overline{\lambda}_0$, then both $\tilde{x}^{SN}_{\lambda,0}$ and $\tilde{x}^{SN}_{\lambda,1}$ are positive. As a result, a narrative in which c is exogenous is the most desirable narrative for the leader.

A.7 Analysis of a Narrative Combining the *SN* and Blame-Shifting

Consider the narrative in which the arrow from y to θ is inverted and the link from θ to c is omitted. We refer to it by the *Strongman Narrative*$'$, *SN*$'$. The factorization formula implies that the subjective probability of high economic performance, conditional on support a, is:

$$\Pr_{SN'}(y=1 \mid a) = \sum_{\theta,c} \Pr(\theta \mid a)\Pr(y=1 \mid \theta)\Pr(c).$$

Plugging in, if $a=1$, this is equal to

$$\Pr(\theta=1 \mid a=1)\Pr(y=1 \mid \theta=1) + \Pr(\theta=0 \mid a=1)\Pr(y=1 \mid \theta=0).$$

When $a = 0$, it is equal to

$$\Pr(\theta = 1 \mid a = 0) \Pr(y = 1 \mid \theta = 1) + \Pr(\theta = 0 \mid a = 0) \Pr(y = 1 \mid \theta = 0).$$

Moreover, the subjective probability of obtaining $c = 1$ is equal to:

$$\Pr_{SN'}(c = 1 \mid a) = \sum_{\theta,y} \Pr(\theta \mid a) \Pr(y \mid \theta) \Pr(c = 1).$$

This is equal to the unconditional probability $\Pr(c = 1)$ for any a.

As a result, following previous calculations, the net expected utility of supporting is equal to:

$$(\alpha_a + \alpha_i \gamma) \frac{\gamma(1-\gamma)(\alpha_y + \alpha_i \beta)}{\Pr(\theta = 1)\Pr(\theta = 0)} + x.$$

Hence, there are thresholds $\tilde{x}_0^{SN} < 0$ and $\tilde{x}_1^{SN} \leq 0$ such that the citizen supports with probability 1 if x is greater than these thresholds. Given that both are (weakly) negative, if $\lambda < 0$, the SN' is more desirable for the leader than either the SN or a narrative in which c is portrayed as exogenous, as depicted in Figure **??**. When $\lambda > 0$, the SN remains the most desirable narrative for the leader.

B Additional Analyses for the Baseline Model

B.1 Additional Data Generating Processes: Misperception vs. Reality

In the main text, we vary narratives, i.e., the beliefs the citizen has about the data generating process, while keeping the actual data generating process fixed. To emphasize the importance of this approach, it is useful to contrast it with a change in the actual data generating process. For simplicity, we consider the cases in which the citizen has rational expectations and in which the citizen believes the *SN*.

B.1.1 Leader Strength Causes High Economic Performance

Contrary to the model in the main text, suppose that $\Pr(\theta = 1 \mid a) = \gamma$ and $\Pr(y = 1 \mid \theta) = \theta$, i.e., the leader is strong with probability γ if the citizen supports and economic performance is good if and only if the leader is strong. As the *SN* specifies, citizen support is the only cause for leader strength, which in turn causes economic performance.

Suppose that the citizen's utility function is $y+ax$ as before, and the citizen believes in the *SN*, which here means that she has rational expectations. Then, the expected utility of supporting the leader is $\gamma + x$ because given support, the leader will be strong with probability γ, and high economic performance is realized with probability 1. In addition, the citizen obtains the ideological benefits x. The expected utility of not supporting the leader is 0 because without support, the leader will be weak and economic performance will be poor. The citizen supports if $x+\gamma \geq 0$: if the ideological benefits are high enough.

This decision rule is very different from the one in which the citizen has incorrect beliefs, as analyzed in the main text. In particular, here, the net expected utility of supporting is *independent* of past behavior and there is always a unique threshold for supporting the leader ($x \geq -\gamma$), i.e., the steady-state probability of support is $\beta^* = \mathbb{1}(x \geq -\gamma)$. We have shown in the main text that when the citizen believes in the *SN* but true data generating process is given by the *BO* DAG, the net expected utility depends on past behavior and there are multiple (personal) equilibria.

B.1.2 Negative Effect of Leader Strength on the Economy

Consider an environment in which in which strong leadership (or: authoritarian rule), θ, has a direct, negative effect on y. Specifically:

$$\Pr(y = 1 \mid \theta) = \gamma g(\theta),$$

with g decreasing. This is the true DGP (i.e., $a \to \theta \to y$), and it is also consistent with the *SN*. For simplicity, let $\Pr(\theta = 1) = a$ so the regime is strong if and only if the citizen supports.

We now describe behavior under these alternative assumptions. When the citizen supports, the expected payoff is $\gamma g(1) + x$ whereas if the citizen does not support, the expected payoff is $\gamma g(0)$. So the citizen supports if

$$x \geq \gamma \left(g(0) - g(1)\right).$$

Relative to the version in the baseline model, the support base shrinks. Moreover, as above, the net expected utility of supporting is *independent* of past behavior and there is always a unique threshold for supporting the leader, so the steady state probability of support is $\beta^* = \mathbb{1}\left(x \geq \gamma \left(g(0) - g(1)\right)\right)$.

B.1.3 Negative Effect of Non-Support on the Economy

Consider the DAG in which $\Pr(y = 1 \mid a) = \gamma h(a)$, with $h(1) > h(0)$ (so h is increasing). Hence, non-support, $a = 0$, has a direct, negative effect on y.

When the citizen has rational expectations, i.e., believes in this expanded *BO* DAG, behavior is as follows. When supporting, the citizen's payoff is $\gamma h(1) + x$ whereas not supporting yields $\gamma h(0)$. Hence, the citizen supports if and only if:

$$x \geq \underbrace{\gamma \left(h(1) - h(0)\right)}_{\text{Economic Costs of Non-Support}}.$$

When the citizen believes in the *SN*, by contrast, the marginal probability of economic performance is:

$$\sum_{\theta} \Pr(\theta \mid a) \Pr(y \mid a, \theta).$$

The citizen expects the regime to be strong with probability $a\gamma h(a)$ and weak with probability $1 - a\gamma h(a)$. Note that support matters directly and indirectly through its effect on the probability that $y = 1$. Conditional on leader strength θ, the citizen expects high economic performance with probability 1 if $\theta = 1$ and with probability $\frac{\gamma h(0)(1-\beta)}{1-\beta\gamma h(1)}$ if

$\theta = 0$. Hence, the expected utility of support is:

$$\gamma h(1)1 + (1-\gamma h(1))\frac{\gamma h(0)(1-\beta)}{1-\beta\gamma h(1)} + x.$$

Not supporting yields simply $\frac{\gamma h(0)(1-\beta)}{1-\beta\gamma h(1)}$. So, the citizen supports if

$$\gamma h(1)\left[1 - \frac{\gamma h(0)(1-\beta)}{1-\beta\gamma h(1)}\right] + x \geq 0$$

or equivalently

$$\text{NE}(\beta) \equiv \gamma h(1)\frac{1-\gamma h(0) - \gamma\beta(h(1)-h(0))}{1-\gamma\beta h(1)} + x \geq 0.$$

Observe that β has competing effect on the net expected utility of support because both the numerator and the denominator are decreasing in β. Nevertheless, the net expected utility is increasing in β. To see this, differentiate NE and simplify to obtain:

$$\frac{\partial \text{NE}}{\partial \beta} = \frac{\gamma^2 h(1)}{[1-\beta\gamma h(1)]^2}\left[h(0)(1-\gamma h(1))\right] > 0.$$

Hence, a similar equilibrium characterization can be obtained as in the baseline model. There is a unique always support equilibrium if $x > -\gamma h(1)\left[1-\gamma h(0)\right]$. There is a unique never support equilibrium if $x < -\gamma h(1)$. Between these thresholds, both equilibria exist, along with a mixed strategy Personal Equilibrium.

B.2 Time-Varying Ideology

Suppose that the citizen's ideological benefit x is redrawn every period according to F, i.e., $x \sim F$ with support $(-1, 1)$. We are looking for a threshold $\hat{x}$ such that the citizen supports if $x \geq \hat{x}$. The probability that the citizen supports is given by $\beta = 1 - F(\hat{x})$.

Given a realized x, the net expected utility of supporting is the same as before:

$$\text{NE}(\beta, x) = x + \frac{\gamma(1-\gamma)}{1-\beta\gamma}.$$

Hence, an equilibrium is characterized by the indifference condition:

$$\hat{x} + \frac{\gamma(1-\gamma)}{1-\gamma\left[1-F(\hat{x})\right]} = 0.$$

Observe that the second term is decreasing in $\hat{x}$.

An equivalent way of looking at the equilibrium condition is to search for an optimal participation rate, β. Since $\beta = 1 - F(\hat{x})$, $\hat{x} = F^{-1}(1-\beta)$. Plugging this in yields:

$$F^{-1}(1-\beta) + \frac{\gamma(1-\gamma)}{1-\beta\gamma} = 0$$

or

$$1 - F\left(-\frac{\gamma(1-\gamma)}{1-\beta\gamma}\right) = \beta.$$

To allow for a more direct comparison with the analysis in the main text, we focus on threshold strategies in the ideological benefits space. Therefore, returning to the indifference condition, define $G(\hat{x}) \equiv \hat{x} + \frac{\gamma(1-\gamma)}{1-\gamma[1-F(\hat{x})]}$. We have $G(-1) = -1 + \gamma < 0$ and $G(1) = 1 + \gamma(1-\gamma) > 0$. Since G is continuous, the Intermediate Value Theorem implies that an equilibrium, $\hat{x}^* \in (-1, 1)$, exists.

In fact, something stronger holds. Recall that $x_0^{SN} = -\gamma$ and $x_1^{SN} = -\gamma(1-\gamma)$ are the thresholds in the complete information case. We have:

$$G(x_0^{SN}) = -\gamma + \frac{\gamma(1-\gamma)}{1-\gamma\left[1-F(-\gamma)\right]} = \frac{-\gamma F(-\gamma)}{1-\gamma\left[1-F(-\gamma)\right]} < 0.$$

and

$$G(x_1^{SN}) = -\gamma(1-\gamma) + \frac{\gamma(1-\gamma)}{1-\gamma\left[1-F(-\gamma(1-\gamma))\right]} = \gamma(1-\gamma)\left[\frac{\gamma F(-\gamma(1-\gamma))}{1-\gamma\left[1-F(-\gamma)\right]}\right] > 0.$$

Hence, again by the Intermediate Value Theorem, there is at least one equilibrium such that $\hat{x}^* \in \left(x_0^{SN}, x_1^{SN}\right)$. Note that all equilibrium thresholds must be negative because $\frac{\gamma(1-\gamma)}{1-\gamma[1-F(\hat{x})]} > 0$. In general, multiple solutions to the indifference condition, or $G(\hat{x}) = 0$, may exist. To see this, consider two examples.

Uniform Distribution Suppose that F is given by the Uniform distribution on $(-1,1)$, so $F(\hat{x}) = \frac{\hat{x}+1}{2}$. Then, the indifference condition can be rearranged to obtain:

$$\left(\frac{\gamma}{2}\right)\hat{x}^2 + \left(\frac{2-\gamma}{2}\right)\hat{x} + \gamma(1-\gamma) = 0.$$

The (potential) solutions are:

$$\hat{x}^*_{1,2} = \frac{-\left(\frac{2-\gamma}{2}\right) \pm \sqrt{\left(\frac{2-\gamma}{2}\right)^2 - 2\gamma^2(1-\gamma)}}{\gamma}.$$

However, only one of these lies in the relevant interval. For example, when $\gamma = 0.8$, the solutions are -1.15 and -0.35, and only the latter corresponds to an equilibrium. This is illustrated in Figure 2.

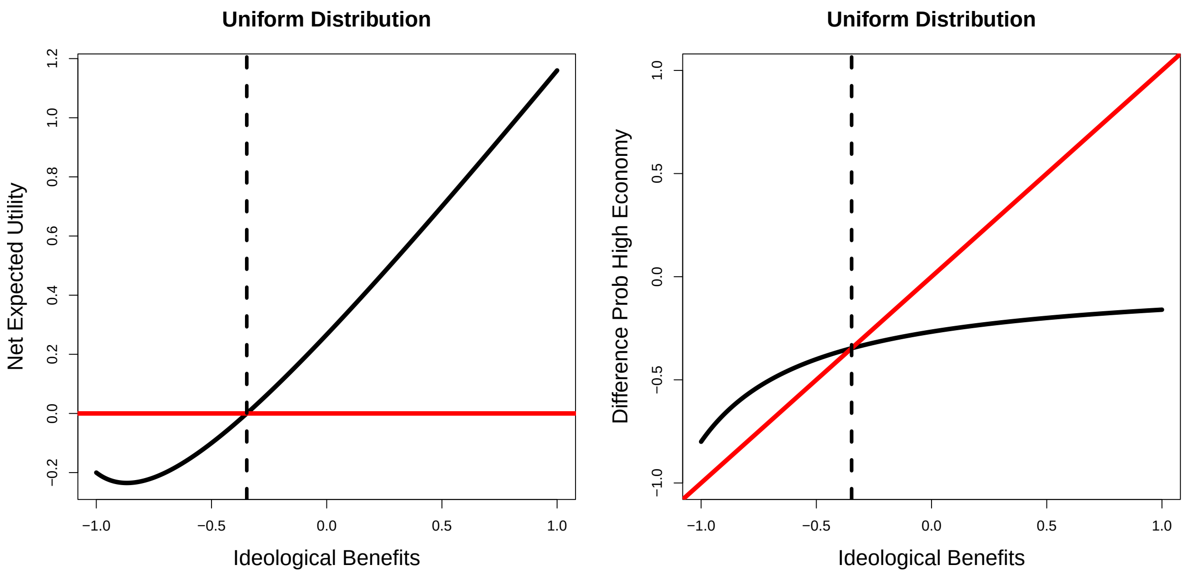

Figure 2: An example when F is the Uniform distribution.

Algebra reveals that this is the case in general:

Result 2. *If F is Uniform on $(-1,1)$, then there is a unique interior equilibrium threshold.*

This equilibrium threshold is also illustrated in Figure 5, left panel, where we graph it as a function of γ. We investigate the comparative static for the general case below.

Normal Distribution Now consider the case in which F corresponds to a truncated Normal distribution, with support $(-1, 1)$. Here, multiple equilibria can exist, see Figure 3, top panel, in which $F = \mathcal{N}(-0.3, 0.2)$. In the left panels, an equilibrium is a root (i.e., solving the condition $G(\hat{x}) = 0$), while in the right panels, an equilibrium is a fixed point.

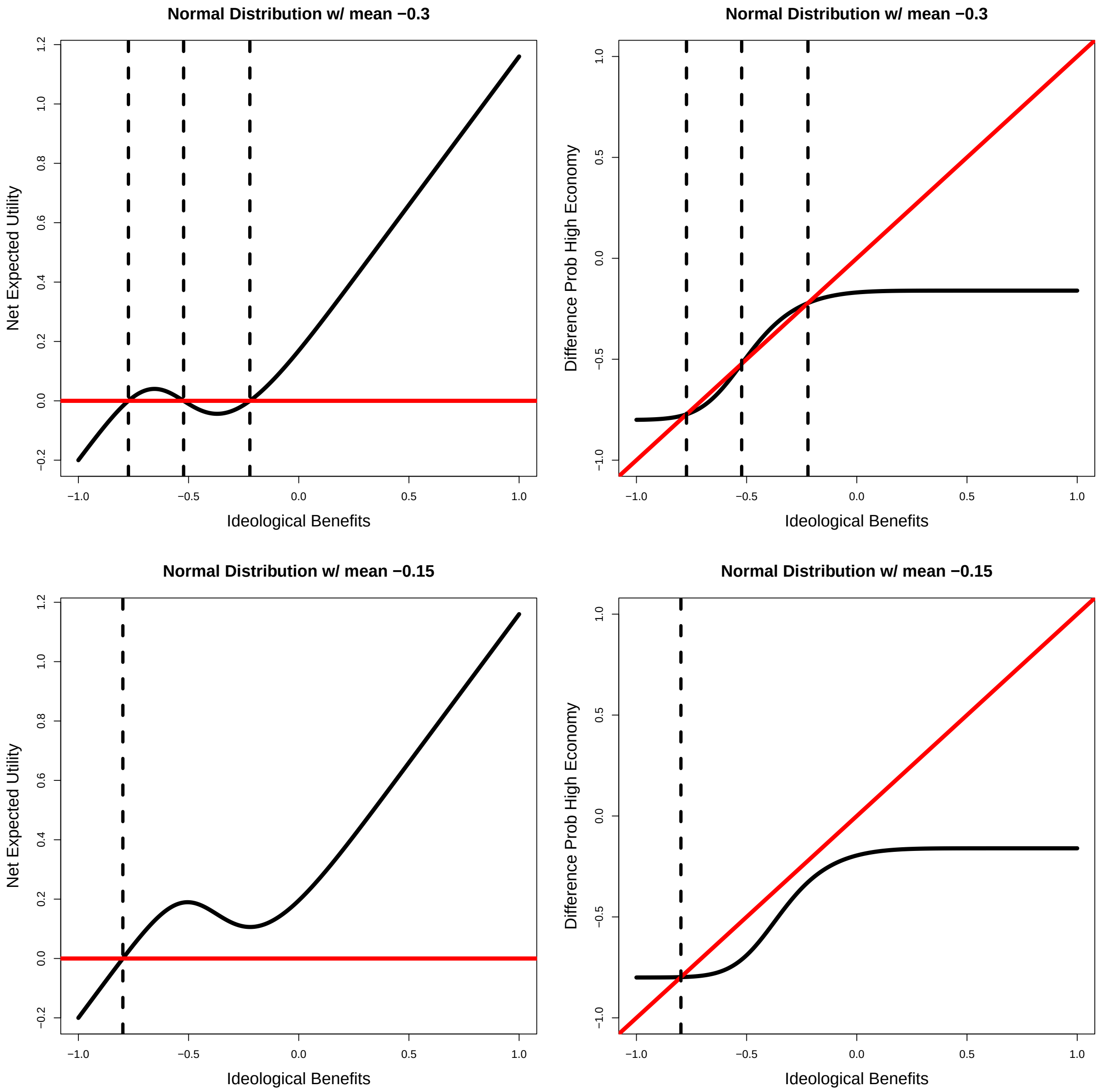

Figure 3: An example with multiple equilibrium thresholds (top panels, $\mu = -0.3$) and a unique equilibrium (bottom panels, $\mu = -0.15$).

Comparison with Baseline Analysis In our benchmark model, we assume that x is known. When the ideological benefits, x, are redrawn every period from the distribution F, it is also possible, although more complicated to construct equilibria with self-fulfilling expectations. What needs to be the case is that there is sufficient probability mass in the "multiplicity region" $(-\gamma, -\gamma(1-\gamma))$. This is illustrated in Figures 3 and 4.

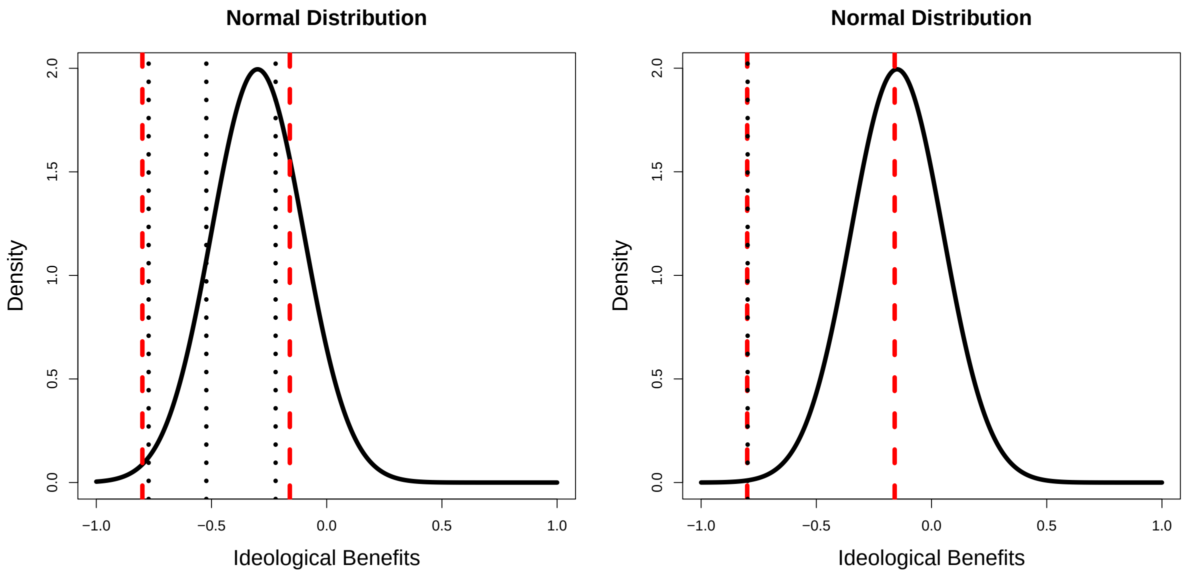

Figure 4: The panels displays the PDFs of ideological benefits (F') along with the critical thresholds $x_0 = -\gamma$ and $x_1 = -\gamma(1-\gamma)$ (in red) and the equilibrium thresholds identified above (in black).

All equilibrium thresholds must be in the multiplicity region, and there are only multiple equilibria if there is sufficient probability mass in this region.

Effect of Increased Likelihood of Good Economic Performance Consider the effect of γ on the equilibrium threshold $\hat{x}^*$, where the threshold is characterized by the indifference condition $G(\hat{x}^*) = 0$. We have:

$$\frac{\partial \hat{x}^*}{\partial \gamma} = -\left.\frac{\frac{\partial G}{\partial \gamma}}{\frac{\partial G}{\partial \hat{x}}}\right|_{\hat{x}^*}.$$

Clearly, $\frac{\partial G}{\partial \hat{x}} > 0$.[2] For the numerator, we have:

$$\frac{\partial G}{\partial \gamma} = \frac{1 - 2\gamma + \gamma^2(1 - F(\hat{x}))}{[1 - \gamma(1 - F(\hat{x})]^2}.$$

[2]When there are multiple equilibrium thresholds, this holds for the stable equilibria, but not the intermediate equilibrium threshold. See Figure 3.

When computing the effect of γ on $\hat{x}^*$, this expression should be evaluated at the equilibrium threshold, $\hat{x} = \hat{x}^*$.

This is somewhat more complicated than in the baseline case because of the presence of the term $\gamma^2 (1 - F(\hat{x}))$. The effect of γ is still ambiguous. Even when there is a unique equilibrium threshold (e.g., when F is the Uniform distribution), the effect of γ on $\hat{x}^*$ can be positive or negative. When there are multiple equilibria, the effect is even more complicated, as the right panel of Figure 5 shows.

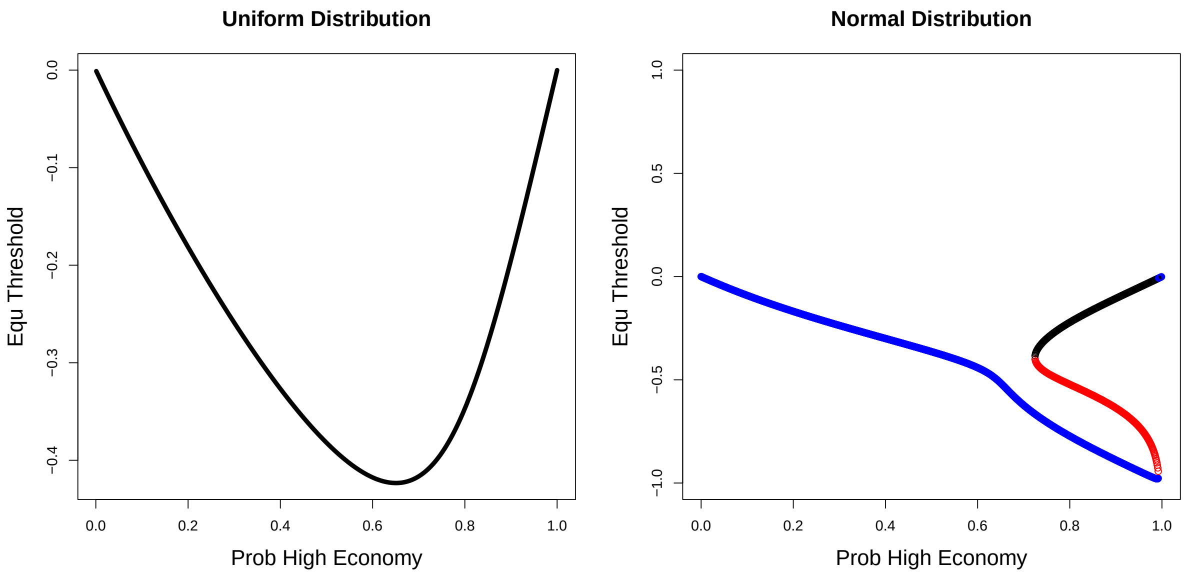

Figure 5: Equilibrium threshold with a Uniform (Normal) distribution in the left (right) panel as a function of the probability that economic performance will be high, γ. In the right panel, multiple equilibria exist for certain values of γ. Then, the blue dots indicate the smallest equilibrium, the red dots are the middle equilibrium, and the black dots are the largest equilibrium.

B.3 Partial Believability of the *SN*

In this section, we explore the behavioral implications of partially believed propaganda. In particular, we assume that with probability δ, the citizen believes in the SN and with probability $1 - \delta$, the citizen has rational expectations. Clearly, the baseline analysis is a special case in which $\delta = 1$. Following Spiegler (2016), for any collection of variables x, the joint distribution is:

$$\Pr_\delta(x) = \delta \Pr_{SN}(x) + (1-\delta)\Pr_{BO}(x)$$

Here, the expected utility of choosing to support, $a = 1$, is:

$$(1-\delta)\gamma + \delta\left[\gamma + (1-\gamma)\frac{(1-\beta)\gamma}{1-\gamma\beta}\right] + x.$$

And the expected utility of choosing $a = 0$ is:

$$(1-\delta)\gamma + \delta\frac{(1-\beta)\gamma}{1-\gamma\beta}.$$

As a consequence, the net expected utility of choosing to protest is:

$$\delta\frac{\gamma(1-\gamma)}{1-\gamma(1-\beta)} + x.$$

This is a generalization of the corresponding expression in the main text. As a consequence, the steady state probability of supporting, β^*, is given by:

$$\begin{aligned}
&\beta^* = 0 && \text{if } x < -\gamma\delta, \\
&\beta^* \in \left\{0, \frac{x+\delta\gamma(1-\gamma)}{x\gamma}, 1\right\} && \text{if } x \in (-\gamma\delta, -(1-\gamma)\gamma\delta), \\
&\beta^* = 1 && \text{if } x > -\gamma(1-\gamma)\delta.
\end{aligned}$$

All quantities have to be adjusted by the probability that the citizen believes that the *SN* is the actual data generating process (δ). However, the fundamental forces that shape the citizen's decision-making remain the same. Define $x_0^\delta = -\delta\gamma$ and $x_1^\delta = -\delta\gamma(1-\gamma)$, and observe that both of these expressions are decreasing in δ. Hence, the leader will find the *SN* to be more desirable according to Definition **??** when the probability that the citizen attaches to the *SN* being the correct DAG is higher.

References

Spiegler, R. (2016). Bayesian networks and boundedly rational expectations. *The Quarterly Journal of Economics 131*(3), 1243–1290.